Cubics related with Concurrent Tangents

Part 2 : Conics

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Abstract

This is a sequel and a generalization of our previous paper. The circles of Part 1 are hereby replaced by conics passing through three fixed points and a vertex of the reference triangle ABC.

1 Theorems and definitions

1.1 The cubics $\mathcal{K}(P_1, P_2, P_3)$, $\mathcal{K}'(P_1, P_2, P_3)$ and $\mathcal{K}''(P_1, P_2, P_3)$

Let $P_1 = p_1 : q_1 : r_1$, $P_2 = p_2 : q_2 : r_2$, $P_3 = p_3 : q_3 : r_3$ be three distinct non collinear points not lying on the sidelines of ABC. Note that these points are not necessarily all real nor finite points : two of them can be imaginary conjugated points and/or points on the line at infinity. In our previous paper, two of the points were the circular points at infinity.

Let M be a variable point and γ_A the conic passing through P_1, P_2, P_3, A and M. γ_B and γ_C are defined similarly.

Let $\mathcal{T}_A$, $\mathcal{T}_B$, $\mathcal{T}_C$ be the tangents at A, B, C to the conics γ_A, γ_B, γ_C respectively.

Theorem 1 (Definition of $\mathcal{K}(P_1, P_2, P_3)$)**.** *The tangents $\mathcal{T}_A$, $\mathcal{T}_B$, $\mathcal{T}_C$ are concurrent (at N) if and only if M lies on a circumcubic $\mathcal{K}(P_1, P_2, P_3)$ passing through P_1, P_2, P_3 or on a sideline of the triangle $P_1P_2P_3$.*

$\mathcal{K}(P_1, P_2, P_3)$ also contains :

- Q_1, Q_2, Q_3 where Q_i is the isoconjugate of P_i in the isoconjugation that swaps P_j and P_k.
- $R_1 = P_2Q_3 \cap P_3Q_2$, R_2 and R_3 similarly.
- the fourth common point S_1 of the two circumconics passing through P_2, Q_3 and P_3, Q_2, S_2 and S_3 similarly.
- the second common point T_1 of the line P_1R_1 and the circumconic passing through P_1, S_1, T_2 and T_3 similarly.
- the third point U_1 of $\mathcal{K}(P_1, P_2, P_3)$ on the line P_2P_3, U_2 and U_3 similarly. When M is one of these points U_i then N is the corresponding point P_i.
- $V_1 = P_2S_3 \cap P_3S_2$, V_2 and V_3 similarly.
- W_1, W_2, W_3 where W_i is the isoconjugate of U_i in the isoconjugation that swaps P_j and P_k.
- $Y_1 = R_2V_3 \cap R_3V_2$, Y_2 and Y_3 similarly.

Theorem 2 (Definition of $\mathcal{K}'(P_1, P_2, P_3)$)**.** *The locus of the common point N of these three tangents is another circumcubic $\mathcal{K}'(P_1, P_2, P_3)$ passing through P_1, P_2, P_3.*

$\mathcal{K}'(P_1, P_2, P_3)$ also contains :

- R'_1, R'_2, R'_3 where R'_i is the crosspoint P_j and P_k i.e. the pole of the line P_jP_k in the circumconic passing through P_j and P_k. R'_i is the homologue of R_i.
- $T'_1 = P_2R'_3 \cap P_3R'_2$, T'_2 and T'_3 similarly.

Theorem 3 (Definition of $\mathcal{K}''(P_1, P_2, P_3)$)**.** *$\mathcal{K}(P_1, P_2, P_3)$ and $\mathcal{K}'(P_1, P_2, P_3)$ generate a pencil of circumcubics passing through P_1, P_2, P_3. This pencil contains one and only one cubic without term in xyz denoted by $\mathcal{K}''(P_1, P_2, P_3)$.*

2 Special cubics

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References

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